

Name: Key

Block: _____

Use prime factorization to answer the following:

1. Determine the greatest common factor of 220 and 860.

$$\begin{array}{r} 2 \overline{)220} \\ 2 \overline{)110} \\ 5 \overline{)55} \\ 11 \end{array}$$

$$\begin{array}{r} 2 \overline{)860} \\ 2 \overline{)430} \\ 5 \overline{)215} \\ 43 \end{array}$$

$$220 = 2 \cdot 2 \cdot 5 \cdot 11$$

$$860 = 2 \cdot 2 \cdot 5 \cdot 43$$

$$\begin{aligned} \text{GCF} &= 2 \cdot 2 \cdot 5 \\ &= 20 \end{aligned}$$

2. Determine the least common multiple of 15, 32, 44

$$\left. \begin{array}{l} 3 \overline{)15} \\ 5 \end{array} \right\} \left. \begin{array}{l} 2 \overline{)32} \\ 2 \overline{)16} \\ 2 \overline{)8} \\ 2 \overline{)4} \\ 2 \end{array} \right\} \left. \begin{array}{l} 2 \overline{)44} \\ 2 \overline{)22} \\ 11 \end{array} \right\}$$

$$15 = 3 \cdot 5$$

$$32 = 2^5$$

$$44 = 2^2 \cdot 11$$

Take each factor
to the highest
power

$$\begin{aligned} \text{LCM} &= 2^5 \cdot 3 \cdot 5 \cdot 11 \\ &= 5280 \end{aligned}$$

3. Show how you would determine the square root of 196 without a calculator.

$$\begin{array}{r} 2 \overline{)196} \\ 2 \overline{)98} \\ 7 \overline{)49} \\ 7 \end{array}$$

$$196 = 2 \cdot 2 \cdot 7 \cdot 7$$

$$196 = (2 \cdot 2) \cdot (7 \cdot 7)$$

$$= (2 \cdot 7) \cdot (2 \cdot 7)$$

$$196 = (14) \cdot (14)$$

$$\therefore \sqrt{196} = 14$$

Basics: Factor the following.

$$1. 12x^2y^3 - 8x^4y^2 + 4x^5y^2$$

$$= 4x^2y^2(3y - 2x^2 + x^3)$$

$$5. x^2 + 5x - 14$$

$$= (x+7)(x-2)$$

$mn = -14$
7 -2

$$2. 5x^2 + 12x + 4$$

$mn = 5(4)$
 $= 20$
10 2

$$= 5x^2 + 10x + 2x + 4$$

$$= 5x(x+2) + 2(x+2)$$

$$= (x+2)(5x+2)$$

$$6. 6x^3 - 2x^2 - 4x$$

$$= 2x(3x^2 - x - 2)$$

$mn = 3(-2)$
 $= -6$
-3 2

$$= 2x[3x^2 - 3x + 2x - 2]$$

$$= 2x[(3x^2 - 3x) + (2x - 2)]$$

$$= 2x[3x(x-1) + 2(x-1)]$$

$$= 2x(x-1)(3x+2)$$

$$3. 9a^2 - 16b^2$$

$$= (3a)^2 - (4b)^2$$

$$= (3a + 4b)(3a - 4b)$$

$$7. x^4 - 3x^2 - 18$$

$mn = -18$
-6 3

$$= (x^2 - 6)(x^2 + 3)$$

notice:

$$4. 25m^2 + 60mn + 36n^2$$

$$(5m)^2 + 2(5m)(6n) + (6n)^2$$

$$a^2 + 2ab + b^2$$

$$= (a+b)^2$$

$$= (5m+6n)^2$$

$$8. 4x^2 + 11xy + 6y^2$$

$mn = 4(6)$
 $= 24$
8 3

$$= 4x^2 + 8xy + 3xy + 6y^2$$

$$= 4x(x+2y) + 3y(x+2y)$$

$$= (x+2y)(4x+3y)$$

or

$$25m^2 + 60mn + 36n^2$$

$mn = 25(36)$
 $= 900$
30 30

$$= 25m^2 + 30mn + 30mn + 36n^2$$

$$= 5m(5m+6n) + 6n(5m+6n)$$

$$= (5m+6n)(5m+6n)$$

Beyond Basics: Factor the following.

9. $16m^4 - 1$

$$\begin{aligned}
 &= (4m^2)^2 - (1)^2 \\
 &= (4m^2 + 1)(4m^2 - 1) \\
 &= (4m^2 + 1)((2m)^2 - (1)^2) \\
 &= (4m^2 + 1)(2m + 1)(2m - 1)
 \end{aligned}$$

10. $4x^4 - 17x^2 + 4$

$$\begin{aligned}
 &= 4x^4 - 16x^2 - 1x^2 + 4 \quad \begin{array}{l} mn = 4(4) \\ = 16 \\ -16 = -1 \end{array} \rightarrow = ((2x)^2 - (1)^2)((x)^2 - (2)^2) \\
 &= 4x^2(x^2 - 4) - 1(x^2 - 4) \\
 &= (4x^2 - 1)(x^2 - 4) \\
 &= (2x + 1)(2x - 1)(x + 2)(x - 2)
 \end{aligned}$$

Expand and Simplify

11. $(3x + y - 1)(2x - 4) - (3x + 2y)^2$

$$\begin{aligned}
 &= 3x(2x - 4) + y(2x - 4) - 1(2x - 4) - (3x + 2y)(3x + 2y) \\
 &= 6x^2 - 12x + 2xy - 4y - 2x + 4 - (9x^2 + 6xy + 6xy + 4y^2) \\
 &= 6x^2 - 14x + 2xy - 4y + 4 - 9x^2 - 12xy - 4y^2 \\
 &= -3x^2 - 14x - 10xy - 4y + 4 - 4y^2
 \end{aligned}$$

Fill in the blanks to create perfect square trinomials:

$$(a + b)^2 = a^2 + 2ab + b^2$$

12. $(x + 5)^2 = x^2 + 10x + 25$

$$\begin{array}{ccc}
 (a + b)^2 & = & a^2 + 2ab + b^2 \\
 \downarrow & \downarrow & \downarrow \\
 x & 5 & x^2 + 2(x)(5) + (5)^2
 \end{array}$$

14. $(x - 6)^2 = x^2 - 12x + 36$

$$\begin{array}{ccc}
 (a - b)^2 & = & a^2 - 2ab + b^2 \\
 (x - \underset{\uparrow 6}{\quad})^2 & = & (x)^2 - 2(x)(\underset{\uparrow 6}{\quad}) + (\underset{\uparrow 6}{\quad})^2 \\
 & & \text{"b" must be 6}
 \end{array}$$

13. $(x - 9)^2 = x^2 - 18x + 81$

$$\begin{array}{ccc}
 (a - b)^2 & = & a^2 - 2ab + b^2 \\
 (x - \underset{\uparrow 9}{\quad})^2 & = & x^2 - 2(x)(\underset{\uparrow 9}{\quad}) + (\underset{\uparrow 9}{\quad})^2 \\
 & & \text{so } b = 9
 \end{array}$$

15. $(2x + 6)^2 = 4x^2 + 24x + 36$

$$\begin{array}{ccc}
 (a + b)^2 & = & a^2 + 2ab + b^2 \\
 (2x + \underset{\uparrow 6}{\quad})^2 & = & (2x)^2 + 2(2x)(\underset{\uparrow 6}{\quad}) + (\underset{\uparrow 6}{\quad})^2 \\
 & & \text{so } b = 6
 \end{array}$$