Name: _Kay_   Block : _____

Perfect
Squares

$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
25
36
49
64
81
100

1. Write in simplified radical form.

**a)** $\sqrt{150}$

$= \sqrt{25 \cdot 6}$

$= \sqrt{25}\sqrt{6}$

$= 5\sqrt{6}$

Perfect
Cubes
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
125

**b)** $\sqrt[3]{135}$

$= \sqrt[3]{27 \cdot 5}$

$= \sqrt[3]{27} \cdot \sqrt[3]{5}$

$= 3\sqrt[3]{5}$

**c)** $\sqrt{112}$

$= \sqrt{4 \cdot 28}$

$= \sqrt{4}\sqrt{28}$

$= 2\sqrt{28}$

$= 2\sqrt{4 \cdot 7}$

$= 2\sqrt{4}\sqrt{7}$

$= 2(2)\sqrt{7}$

$= 4\sqrt{7}$

OR

$\sqrt{112} = \sqrt{16 \cdot 7}$

$= \sqrt{16}\sqrt{7}$

$= 4\sqrt{7}$

**d)** $\sqrt[4]{162}$

$= \sqrt[4]{81 \cdot 2}$

$= \sqrt[4]{81} \sqrt[4]{2}$

$= 3\sqrt[4]{2}$

Perfect
$x^4$

$2^4 = 16$
$3^4 = 81$
$4^4 = 256$

2. Write as an entire radical.

**a)** $6\sqrt{5}$

$= \sqrt{6 \cdot 6}\sqrt{5}$

$= \sqrt{36}\sqrt{5}$

$= \sqrt{180}$

**b)** $3\sqrt{14}$

$= \sqrt{3 \cdot 3}\sqrt{14}$

$= \sqrt{3 \cdot 3 \cdot 14}$

$= \sqrt{126}$

**c)** $4\sqrt[3]{3}$

$= \sqrt[3]{4 \cdot 4 \cdot 4}\sqrt[3]{3}$

$= \sqrt[3]{64}\sqrt[3]{3}$

$= \sqrt[3]{64 \cdot 3}$

$= \sqrt[3]{192}$

**d)** $2\sqrt[4]{2}$

$= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2}\sqrt[4]{2}$

$= \sqrt[4]{16}\sqrt[4]{2}$

$= \sqrt[4]{16 \cdot 2}$

$= \sqrt[4]{32}$

3. Express each power as a radical.

**a)** $12^{\frac{1}{4}}$

$= \sqrt[4]{12}$

**b)** $(-50)^{\frac{5}{3}}$

$= \sqrt[3]{(-50)^5}$

**c)** $1.2^{0.5}$

$= 1.2^{\frac{1}{2}}$

$= \sqrt{1.2}$

**d)** $\left(\frac{3}{8}\right)^{\frac{1}{3}}$

$= \sqrt[3]{\frac{3}{8}}$

4. Express each radical as a power.

**a)** $\sqrt{1.4}$

$= 1.4^{\frac{1}{2}}$

**b)** $\sqrt[3]{13^2}$

$= 13^{\frac{2}{3}}$

**c)** $\left(\sqrt[5]{2.5}\right)^4$

$= 2.5^{\frac{4}{5}}$

**d)** $\left(\sqrt[4]{\frac{2}{5}}\right)^3$

$= \left(\frac{2}{5}\right)^{3/4}$

5. Evaluate each power without using a calculator.

a) $16^{0.25}$

$= 16^{\frac{1}{4}}$

$= \sqrt[4]{16}$

$= 2$

b) $1.44^{\frac{1}{2}}$

$= \sqrt{1.44}$

$= 1.2$

c) $(-8)^{\frac{5}{3}}$

$= \sqrt[3]{(-8)^5}$

$= \left(\sqrt[3]{-8}\right)^5$

$= (-2)^5$

$= -32$

d) $\left(\frac{9}{16}\right)^{\frac{3}{2}}$

$= \sqrt{\left(\frac{9}{16}\right)^3}$

$= \frac{\left(\sqrt{9}\right)^3}{\left(\sqrt{16}\right)^3}$

$= \frac{3^3}{4^3} = \frac{9}{64}$

6. Evaluate each power without using a calculator.

a) $\frac{2^{-2}}{1}$

$= \left(\frac{1}{2}\right)^2$

$= \frac{1^2}{2^2}$

$= \frac{1}{4}$

b) $\left(\frac{2}{3}\right)^{-3}$

$= \left(\frac{3}{2}\right)^3$

$= \frac{3^3}{2^3}$

$= \frac{27}{8}$

c) $\left(\frac{4}{25}\right)^{-\frac{3}{2}}$

$= \left(\frac{25}{4}\right)^{\frac{3}{2}}$

$= \sqrt{\left(\frac{25}{4}\right)^3}$

$= \frac{\left(\sqrt{25}\right)^3}{\left(\sqrt{4}\right)^3}$

$\rightarrow = \frac{(5)^3}{(4)^3}$

$= \frac{125}{64}$

7. a) A company designs a container with the shape of a triangular prism to hold 500 mL of juice. The bases of the prism are equilateral triangles with side length $s$ centimetres. The height, $h$ centimetres, of the prism is given by the formula:

$h = 2000(3)^{-\frac{1}{2}}s^{-2}$

What is the height of a container with base side length 8.0 cm? Give your answer to the nearest tenth of a centimetre.

$h = 2000(3)^{-\frac{1}{2}}(8)^{-2}$

$= 18 \text{cm}$

b) Radioactive isotopes decay. The half-life of an isotope is the time for its mass to decay by $\frac{1}{2}$. For example, polonium-210 has a half-life of 20 weeks. So, a sample of 100 g would decay to 50 g in 20 weeks. The percent, $P$, of polonium remaining after time $t$ weeks is given by the formula $P = 100(0.5)^{\frac{t}{20}}$. What percent of polonium remains after 30 weeks?

$P = 1000(0.5)^{\frac{30}{20}}$

$= 353.55\%$

8. Evaluate each power without using a calculator.

**a)** $81^{0.75}$

$= 81^{3/4}$

$= \left(\sqrt[4]{81}\right)^3$

$= 3^3$

$= 27$

**b)** $\left(\dfrac{36}{49}\right)^{\frac{3}{2}}$

$= \sqrt{\left(\dfrac{36}{49}\right)^3}$

$= \dfrac{\sqrt{36}^3}{\sqrt{49}^3}$

$= \dfrac{6^3}{7^3} = \dfrac{216}{343}$

**c)** $(-0.027)^{\frac{5}{3}}$

$= \sqrt[3]{(-0.027)}^5$

$= (-0.3)^5$

$= -0.00243$

**d)** $\left(\dfrac{4}{9}\right)^{-2}$

$= \left(\dfrac{9}{4}\right)^2$

$= \dfrac{9^2}{4^2}$

$= \dfrac{81}{16}$

**e)** $16^{-\frac{3}{4}}$

$= \left(\dfrac{1}{16}\right)^{3/4}$

$= \sqrt[4]{\left(\dfrac{1}{16}\right)^3}$

$= \left(\dfrac{1}{2}\right)^3$

$= \dfrac{1^3}{2^3} = \dfrac{1}{8}$

**f)** $\left(\dfrac{25}{64}\right)^{-\frac{3}{2}}$

$= \left(\dfrac{64}{25}\right)^{3/2}$

$= \sqrt{\left(\dfrac{64}{25}\right)^3}$

$= \left(\dfrac{8}{5}\right)^3$

$= \dfrac{8^3}{5^3} = \dfrac{512}{125}$

**g)** $243^{0.6}$

$= 243^{\frac{6}{10}}$

$= 243^{3/5}$

$= \sqrt[5]{243}^3$

$= 3^3$

$= 27$

**h)** $(-0.064)^{-\frac{2}{3}}$

$= \left(\dfrac{1}{-0.064}\right)^{\frac{2}{3}}$

$= \sqrt[3]{\left(\dfrac{1}{-0.064}\right)^2}$

$= \left(\dfrac{1}{-0.4}\right)^2$

$= \dfrac{1^2}{(-0.4)^2}$

$= \dfrac{1}{0.16}$

**i)** $\left(\dfrac{49}{121}\right)^{-\frac{3}{2}}$

$= \left(\dfrac{121}{49}\right)^{3/2}$

$= \sqrt{\left(\dfrac{121}{49}\right)^3}$

$= \left(\dfrac{11}{7}\right)^3$

$= \dfrac{11^3}{7^3} = \dfrac{1331}{343}$

9. Simplify

**a)** $(3m^4n)^2$

$= 3^2(m^4)^2 n^2$

$= 9 m^8 n^2$

**b)** $\left(\dfrac{x^2 y}{y^{-2}}\right)^{-2}$

$= (x^2 y \, y^2)^{-2}$

$= (x^2 y^3)^{-2}$

$= \left(\dfrac{1}{x^2 y^3}\right)^2$

$= \dfrac{1}{x^4 y^6}$

**c)** $(16\,a^2 b^6)^{-\frac{1}{2}}$

$= \left(\dfrac{1}{16 a^2 b^6}\right)^{\frac{1}{2}}$

$= \dfrac{1^{\frac{1}{2}}}{16^{\frac{1}{2}}(a^2)^{\frac{1}{2}}(b^6)^{\frac{1}{2}}}$

$= \dfrac{1}{\sqrt{16}\,a^{\frac{2}{2}}\,b^{\frac{6}{2}}}$

$= \dfrac{1}{4\,a\,b^3}$

**d)** $\left(\dfrac{r^3 s^{-1}}{s^{-2} r^{-2}}\right)^{-\frac{2}{3}}$

$= \left(\dfrac{r^3 s^2 r^2}{s^1}\right)^{-\frac{2}{3}}$

$= (r^5 s)^{-\frac{2}{3}}$

$= \left(\dfrac{1}{r^5 s}\right)^{\frac{2}{3}}$

$= \dfrac{1}{(r^5)^{\frac{2}{3}}(s)^{\frac{2}{3}}} = \dfrac{1}{r^{\frac{10}{3}} s^{\frac{2}{3}}}$

10. Simplify

**a)** $(a^3 b)(a^{-1} b^4)$

$= a^{3+-1} b^{1+4}$

$= a^2 b^5$

**b)** $\left(x^{\frac{1}{2}} y\right)\left(x^{\frac{3}{2}} y^{-2}\right)$

$= x^{\frac{1}{2}+\frac{3}{2}} y^{1+-2}$

$= x^{\frac{4}{2}} y^{-1}$

$= \dfrac{x^2}{y}$

**c)** $\dfrac{a^3}{a^5} \cdot a^{-3}$

$= a^{3-5} \cdot a^{-3}$

$= a^{-2} \cdot a^{-3}$

$= a^{-5}$

$= \dfrac{1}{a^5}$

**d)** $\dfrac{x^2 y}{x^{\frac{1}{2}} y^{-2}}$

$= x^{2-\frac{1}{2}} y^{1--2}$

$= x^{\frac{4}{2}-\frac{1}{2}} y^{1+2}$

$= x^{\frac{3}{2}} y^3$

11. Simplify, then evaluate.

**a)** $\left(\dfrac{3}{2}\right)^{\frac{3}{2}} \cdot \left(\dfrac{3}{2}\right)^{\frac{1}{2}}$

$= \left(\dfrac{3}{2}\right)^{\frac{3}{2}+\frac{1}{2}}$

$= \left(\dfrac{3}{2}\right)^{\frac{4}{2}}$

$= \left(\dfrac{3}{2}\right)^2$

$= \dfrac{3^2}{2^2}$

$= \dfrac{9}{4}$

**b)** $\dfrac{(-5.5)^{\frac{2}{3}}}{(-5.5)^{-\frac{4}{3}}}$

$= (-5.5)^{\frac{2}{3}--\frac{4}{3}}$

$= (-5.5)^{\frac{6}{3}}$

$= (-5.5)^2$

$= 30.25$

**c)** $\left[\left(-\dfrac{12}{5}\right)^{\frac{1}{3}}\right]^{\frac{6}{1}}$

$= \left(-\dfrac{12}{5}\right)^{\frac{6}{3}}$

$= \left(-\dfrac{12}{5}\right)^2$

$= \dfrac{144}{25}$

**d)** $\dfrac{0.16^{\frac{3}{4}}}{0.16^{\frac{1}{4}}}$

$= 0.16^{\frac{3}{4}-\frac{1}{4}}$

$= 0.16^{\frac{2}{4}}$

$= 0.16^{\frac{1}{2}}$

$= \sqrt{0.16}$

$= 0.4$

12. Simplify, then evaluate.

**a)** $\left(\dfrac{2}{5}\right)^{1.5}\left(\dfrac{2}{5}\right)^{0.5}$

$= \left(\dfrac{2}{5}\right)^{1.5+0.5}$

$= \left(\dfrac{2}{5}\right)^{2}$

$= \dfrac{2^2}{5^2}$

$= \dfrac{4}{25}$

**b)** $\dfrac{0.25^{-\frac{2}{3}}}{0.25^{-\frac{5}{3}}}$

$= 0.25^{-\frac{2}{3}--\frac{5}{3}}$

$= 0.25^{-\frac{2}{3}+\frac{5}{3}}$

$= 0.25^{3/3}$

$= 0.25^{1}$

**c)** $\dfrac{\left(0.36^{\frac{5}{2}}\right)\left(0.36^{\frac{3}{2}}\right)}{0.36^{\frac{9}{2}}}$

$= \dfrac{0.36^{\frac{5}{2}+\frac{3}{2}}}{0.36^{9/2}}$

$= \dfrac{0.36^{8/2}}{0.36^{9/2}}$

$= 0.36^{\frac{8}{2}-\frac{9}{2}}$

$= 0.36^{-\frac{1}{2}}$

$= \dfrac{1}{0.36^{1/2}}$

$= \dfrac{1}{\sqrt{0.36}}$

$= \dfrac{1}{0.6} = \dfrac{10}{6} = \dfrac{5}{3}$

**d)** $\dfrac{\left(-\dfrac{1}{8}\right)^{\frac{7}{3}}\left(-\dfrac{1}{8}\right)^{\frac{2}{3}}}{\left(-\dfrac{1}{8}\right)^{\frac{5}{3}}\left(-\dfrac{1}{8}\right)}$

$\dfrac{\left(-\dfrac{1}{8}\right)^{\frac{7}{3}+\frac{2}{3}}}{\left(-\dfrac{1}{8}\right)^{\frac{5}{3}-\frac{3}{3}}}$

$= \dfrac{\left(-\dfrac{1}{8}\right)^{\frac{9}{3}}}{\left(-\dfrac{1}{8}\right)^{\frac{2}{3}}}$

$= \left(-\dfrac{1}{8}\right)^{\frac{9}{3}-\frac{2}{3}}$

$= \left(-\dfrac{1}{8}\right)^{\frac{7}{3}}$

$= \sqrt[3]{\left(-\dfrac{1}{8}\right)^{7}}$

$= \left(-\dfrac{1}{3}\right)^{7}$

$= \dfrac{-1}{2189}$

13. Simplify.

**a)** $\dfrac{(a^{-2}b^{-1})^{-3}}{a^3b}$

$= \dfrac{(a^{-2})^{-3}(b^{-1})^{-3}}{a^3b}$

$= \dfrac{a^6 b^3}{a^3 b}$

$= a^{6-3}b^{3-1}$

$= a^3 b^2$

**b)** $\left(\dfrac{2x^{-4}y^{-3}}{4x^2y^{-5}}\right)^{-4}$

$= \left(\dfrac{2}{4}\dfrac{y^5}{x^4x^2y^3}\right)^{-4}$

$= \left(\dfrac{1y^2}{2x^6}\right)^{-4}$

$= \left(\dfrac{2x^6}{y^2}\right)^{4}$

$= \dfrac{16x^{24}}{y^8}$

**c)** $\dfrac{-15a^{-\frac{1}{2}}b}{5ab^{-\frac{3}{2}}}$

$= -3a^{-\frac{1}{2}-1}b^{1--\frac{3}{2}}$

$= -3a^{-\frac{1}{2}-\frac{2}{2}}b^{\frac{2}{2}+\frac{3}{2}}$

$= -3a^{-\frac{3}{2}}b^{\frac{5}{2}}$

$= \dfrac{-3b^{5/2}}{a^{3/2}}$

**d)** $\left(\dfrac{x^6z^{-\frac{1}{3}}}{-125y^{-9}z^{\frac{8}{3}}}\right)^{-\frac{1}{3}}$

$= \left(\dfrac{x^6 y^9}{-125 z^{\frac{1}{3}}z^{\frac{8}{3}}}\right)^{-\frac{1}{3}}$

$= \left(\dfrac{-125 z^{\frac{9}{3}}}{x^6 y^9}\right)^{\frac{1}{3}}$

$= \dfrac{(-125)^{\frac{1}{3}}\left(z^{\frac{9}{3}}\right)^{\frac{1}{3}}}{(x^6)^{\frac{1}{3}}(y^9)^{\frac{1}{3}}}$

$= \dfrac{\sqrt[3]{-125}\ z^{9/9}}{x^{\frac{6}{3}}y^{9/3}}$

$= \dfrac{-5z}{x^2 y^3}$